D1AST2402 **(2 Pages)** **Name............................**
Reg.No.........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C02-ANALTYCAL TOOL FOR STATISTICS I

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define continuity and uniform continuity of a function.
2. If A is a set in a metric space, show that its derived set (A'), and its closure $(\bar{A})$ are closed sets.
3. Define a perfect set. Give one example for a perfect set.
4. If $f(x) = \begin{cases} 1 & if \quad x \text{ is rational} \\ 0 & if \quad x \text{ is irrational} \end{cases}$, discuss on the continuity of this function.
5. Define a partition of an interval and its refinement. Give example.
6. If f(x) is Riemann-Stieltjes integral with respect to $\alpha(x)$($f \in R(\alpha)$), on [a,b], show that $|f| \in R(\alpha)$ on [a,b].
7. Define point wise and uniform convergence of a series of functions.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Define a countable set. Show that the set Q of rational numbers is countable.
9. Let $f: R \rightarrow R$ be given by $f(x) = \begin{cases} a & if \quad x < 0 \\ ax^2 + 2x + 5 & if \quad x \geq 0 \end{cases}$. What value of 'a' ensures the continuity of f(x) at x=0? Is it differentiable at x=0, for that value of a?
10. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on [a,b], show that $fg \in R(\alpha)$.
11. Define a compact set. Show that if K is a compact subset of R, then K is closed and bounded.
12. State and prove a necessary and sufficient condition for the existence of Riemann- Stieltjes integral of f(x) with respect to $\alpha(x)$ on $[a, b]$, where f(x) and $\alpha(x)$ are bounded functions on [a,b] and α is monotonic increasing on $[a, b], b \geq a$.
13. State and prove Cauchy's criterion for uniform convergence of a sequence of functions.
14. If $f_n(x) = \frac{x}{1+nx}$ for $x \in [0, \infty)$, discuss the convergence of $f_n(x)$.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Show that finite union of closed sets is closed, and finite intersection of open sets is open.

16. a) State and prove the mean value theorem.

    b) Verify whether $f(x) = x^2 + 2x - 8$ defined on [-4,4] satisfies Lagrange's mean value theorem.

17. a) Assume α is increasing on [a,b]. If P' is finer than P, show that $U(P', f, \alpha) \leq U(P, f, \alpha)$.

    b) Establish that Riemann-Stieltjes integration reduces to Riemann integral.

18. Test for the uniform convergence of the series $\sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2}$ using Weirstrass M test.

(2 × 5 = 10 weightage)