

**FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024**  
**(Regular/Improvement/Supplementary)**

**STATISTICS**

**FMST1C01-MEASURE THEORY AND INTEGRATION**

**Time: 3 Hours**

**Maximum Weightage: 30**

**Part A: Answer any *four* questions. Each carries *two* weightage.**

1. Define: i)  $\sigma$  – field; (ii) measurable space.
2. Explain measure with an example.
3. State Minkowski's inequality.
4. Write a short note on normed linear spaces.
5. Distinguish between positive, negative and null sets with respect to a charge defined on a  $\sigma$  – field.
6. State Caratheodory extension theorem.
7. Write a short note on product measures.

**(4 × 2 = 8 weightage)**

**Part B: Answer any *four* questions. Each carries *three* weightage.**

8. State and prove monotone convergence theorem.
9. Let  $f$  and  $g$  be measurable real valued functions and let  $c$  be a real number. Then show that  $cf, f^2, f + g, fg, |f|$  are also measurable.
10. State and prove Lebesgue dominated convergence theorem.
11. Briefly describe  $L_p$  spaces.
12. State and prove Jordan decomposition theorem.
13. State Radon Nikodym theorem. Prove any one property of it.
14. State and prove monotone class lemma.

**(4 × 3 = 12 weightage)**

**Part C: Answer any *two* questions. Each carries *five* weightage.**

15. State and prove Holder's inequality.
16. If  $f$  is a nonnegative function in  $M(X, \mathbb{R})$ , then show that there exists a non-negative non-decreasing sequence of simple functions  $\{\varphi_n\}$  such that  $f(x) = \lim \varphi_n(x)$  for each  $x \in X$ .
17. State and prove Hahn- decomposition theorem.
18. State and prove Fubini's theorem.

**(2 × 5 = 10 weightage)**