D1APH2402 **(2 Pages)** **Name.....**
Reg.No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)

PHYSICS
FPHY1C02-MATHEMATICAL PHYSICS I

Time: 3 Hours **Maximum Weightage: 30**

Part A: Short answer questions. Answer *all* questions. Each carries *one* weightage

1. Show that the dot product of two vectors is invariant under rotation of coordinates.
2. Derive the expression for the volume element dV in cylindrical coordinates.
3. What is the difference between Hermitian and unitary matrices?
4. Explain the concept of a dual tensor with an example.
5. What is meant by a self-adjoint differential equation? Give an example.
6. What is the Dirac delta function, and where is it applied in physics?
7. Show the relationship between the Beta and Gamma functions.
8. Define the Fourier transform and its inverse.

(8 × 1 = 8 weightage)

Part B: Essay questions. Answer any *two* questions. Each carries *five* weightage

9. Derive the expression for the Laplacian operator in spherical polar coordinates and apply it to solve Laplace's equation for the electrostatic potential due to a point charge.
10. Prove that the eigenvalues of Hermitian matrices are real and that the eigenvectors corresponding to distinct eigenvalues are orthogonal. Illustrate this with an example matrix.
11. Solve the differential equation $(x^2-1)y''+xy'-\lambda y= 0$ using the Frobenius method around x= 1. Discuss the nature of the singular points and the convergence of the series solution.
12. Using the generating function of Legendre polynomials, derive the recurrence relations for the Legendre polynomials $P_n(x)$. Prove the orthogonality of these polynomials over the interval $[-1,1]$.

(2 × 5 = 10 weightage)

(P.T.O.)

Part C: Problems. Answer any *four* questions. Each carries *three* weightage

13. Explain the method of separation of variables in solving second-order differential equations.
14. Define the Gamma function and show how it can be used to evaluate integrals of the form $\int_o^\infty x^{n-1}e^x\ dx$, where n is a positive integer.
15. Write the generating function for Hermite polynomials and use it to prove the recurrence relation $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$.
16. Write the differential equation for spherical Bessel functions and show how it reduces to the standard Bessel equation. Discuss one physical context where spherical Bessel functions arise.
17. Find the Laplace transform of $t^n e^{-at}$ and discuss its properties.
18. Find the Fourier series of the function $f(x)=x^2$ in the interval $0 \leq x \leq 2\pi$. Discuss the importance of Fourier series in solving physical problems.
19. Using the convolution theorem, find the Fourier transform of the product of two functions $f(x)=e^{-ax}$ and $g(x)=e^{-bx}$, where a and b are constants.

(4 × 3 = 12 weightage)