D1AMT2404 **(2 Pages)** **Name.............................**
Reg.No...........................

FIRST SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)

MATHEMATICS
FMTH1C04- DISCRETE MATHEMATICS

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries 1 weightage.

1. Define a partial order relation on a set X. Give example.
2. Let $(X, +, ., ')$ be a Boolean algebra. Prove that for x, y, z in $X, x + x.y = x$ and $x.(x + y) = x$.
3. State and prove the first theorem of graph theory.
4. Let G be a graph with n vertices and m edges. Assume that each vertex of G is of degree either k or $k + 1$. Show that the number of vertices of degree k in G is $(k + 1)n - 2m$.
5. Prove or disprove: No loop can belong to an edge cut.
6. Show that the Peterson graph is nonplanar.
7. Define the star-closure of a language.
8. Find the concatenation of the strings $u = a_1a_2 \dots \dots a_n$ and $v = b_1b_2 \dots \dots b_m$.

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries 2 weightage.

Unit 1

9. Let $(X, \leq)$ be a poset and A a nonempty finite subset of X. Then show that A has atleast one maximal element.
10. i) Define Boolean algebra and subalgebra. Give examples.

    ii) Let $(X, +, ., ')$ be a Boolean algebra and let Y be a subalgebra of X. Then prove that Y with the induced operations is a Boolean algebra with same identity elements in X.
11. Let $(X, +, ., ')$ be a finite Boolean algebra. Then prove that every element of X can be uniquely expressed as a sum of atoms.

(P.T.O.)

Unit 2

12. i) Show that the number of edges of a simple graph with n vertices and ω components is $\frac{(n-\omega)(n-\omega+1)}{2}$.

    ii) Check whether if G is a simple graph with $\delta \geq \frac{n-2}{2}$, then G is connected. Justify.

13. Prove that in a connected graph G with atleast 3 vertices, any two longest paths have a vertex in common.

14. State and prove a characterization of a cut vertex of G.

Unit 3

15. i) Define a regular language.

    ii) Show that the language $\mathrm{L} = \{\mathrm{awa}: \mathrm{w} \in \{a, b\} *\}$ is regular.

16. Find a dfa that recognizes the set of all strings on $\Sigma = \{a, b\}$.

17. i) Define the language generated by a grammar G.

    ii) Find a grammar that generates $L = \{a^n b^{n+1}: n \geq 0\}$.

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage.

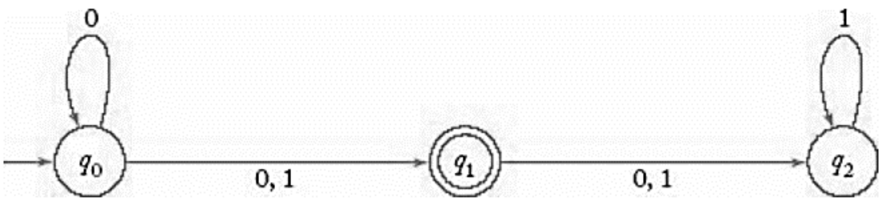
18. Prove that every finite Boolean algebra is isomorphic to a power set Boolean algebra.

19. Prove that a graph is bipartite if and only if it contains no odd cycles.

20. i) Define connectivity and edge connectivity of a graph G.

    ii) Prove that for any loopless connected graph $G, \kappa(G) \leq \lambda(G) \leq \delta(G)$.

21. Convert the following nfa into an equivalent dfa.



(2 × 5 = 10 weightage)