D1AST2305 **(2 Pages)** **Name..........................**
Reg.No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C05-DISTRIBUTION THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Find E(X) if the distribution function of X is F (x) =1 $-$ (1+ x) e^{-x}, x $>$ 0.
2. Let X has a geometric distribution with parameter θ. Prove that $M_x(t) = \frac{\theta e^t}{1-e^t(1-\theta)}$.
3. Stating the conditions, prove convergence of binomial distribution to a Poisson distribution.
4. Define Gamma distribution. For the Gamma distribution with parameters (α, β), prove that $\mu_r^1 = \frac{\Gamma(\alpha+r)}{\Gamma\alpha}\beta, r =$ 1, 2, 3 ....
5. Explain lognormal distribution. What are its important characteristics?
6. Define a bivariate normal distribution. If (X,Y) has a bivariate normal distribution, find the conditional expectation E(Y/X).
7. Define sampling distribution. Establish the inter–relationship between t, x^2 and F distributions.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Define power series distributions. Obtain the recurrence relation for cumulants of power series distributions.
9. Define normal distribution. Derive the moment generating function of normal distribution. Hence find the moment generating function of standard normal distribution.
10. Explain 'lack of memory property' of exponential distribution. Let X_1 and X_2 be two independent r.v.s each having exponential distribution with mean 1.
    Find: P($X_1 < X_2$) and P($X_1 < 2 - X_2$).
11. Explain mixtures of binomial distributions.
12. Define chi-square and derive its distribution. Obtain the mean, variance and moment generating function of chi-square distribution.
13. Derive F distribution. Explain its properties and applications.

(P.T.O.)

14. If f(x, y) = $\frac{x+y}{21}$, x = 1, 2, 3; y = 1, 2

= 0, elsewhere.

Find:

i) the marginal distributions of X and Y.

ii) E(Y/X) and E(X/Y)

iii) The correlation coefficient between X and Y.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Show that binomial distribution tends to Normal distribution under certain conditions to be stated.

16. Explain the origin of Pearsonian system of distributions. Also establish the standard form of Pearsonian Main Type I distribution.

17. In bivariate transformations, obtain the distribution of the sum of two random variables. Let (X, Y) be a two dimensional non-negative continuous r.v having the joint density function

$$f(x, y) = \begin{cases} 4xye^{-(x^2+y^2)} & ; \quad X \geq 0, Y \geq 0 \\ 0 & ; \quad \text{else where} \end{cases}$$

Prove that the density function of $u = \sqrt{X^2 + Y^2}$ is

$$h(u) = 2u^3 e^{-u^2}; \quad 0 \leq u < \infty$$
$$= 0 \qquad ; \text{elsewhere}$$

18. Define non-central ‘t’ distribution. Also derive the pdf of non-central ‘t’ distribution.

(2 × 5 = 10 weightage)