D1AST2303 **(2 pages)** **Name.............................**
Reg.No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C03-ANALYTICAL TOOL FOR STATISTICS II

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Explain Gram Schmidt Orthogonalization process with an example.
2. When do you say that a set of vectors are linearly independent? Check the independence of the vectors (3,1,2), (5,2,3), (2,3,-1).
3. Define subspace of a vector space.
   Examine whether $S = \{(x_1, x_2, x_3): 2x_1 + x_2 + x_3 = 1\}$ is a subspace of $\mathbb{R}^3$.
4. Let A be an idempotent matrix of order $n \times n$. Then prove or disprove:
   Rank (A) + Rank $(I - A) = n$, where I is the unit matrix.
5. Define nilpotent matrix. Show that $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is nilpotent of index three.
6. A mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by
   $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + x_2 + x_3, x_1 + 2x_2 + x_3)$.
   Show that T is a linear mapping. Find the dimension of kernel and image of T.
7. Find the minimal polynomial of $\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Find a basis and dimension of the solution space W of the homogeneous system

   $x + 2y - 2z + 2s - t = 0, \; x + 2y - z + 3s - 2t = 0, \; 2x + 4y - 7z + s + t = 0.$

9. Find the rank. Also find a basis for row space and column space for $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$.

10. Let V be a vector space over the field F. Let U and W are subspaces of V. Show that $\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$.

11. Define an inner product space. Show that every finite dimensional inner product space has an orthonormal basis.

(P.T.O.)

12. Explain the spectral decomposition of a matrix. Find the spectral decomposition of $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & 1 & 2 \end{bmatrix}$.

13. State and prove Rank nullity theorem.

14. Classify the quadratic form $x^2 + 20y^2 + 10z^2 - 4yz - 16zx - 36xy$.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. a) Prove that the modulus of characteristic root of a unitary matrix is unity.
    b) Show that a characteristic root of a skew symmetric matrix is either zero or a purely imaginary number.
16. Show that the following properties hold in a vector space V of dimension n:
    a) Any subset of V which contains more than n elements must be linearly dependent.
    b) Any linearly independent subset of V may be extended to a basis of V.
17. a) Explain Moore Penrose inverse of a matrix. Show that it is unique.
    b) Find the Moore Penrose inverse of $\begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$.
18. a) State and prove a necessary and sufficient condition that a real quadratic form $X'AX$ is positive definite.
    b) Explain signature of a quadratic form $X'AX$ with an example.

(2 × 5 = 10 weightage)