D1AST2302 **(2 Pages)** **Name............................**
Reg.No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C02-ANALTYCAL TOOL FOR STATISTICS I

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Show that every infinite subset of a countable set is countable.
2. Define a metric space. Illustrate with an example.
3. Define limit and continuity of functions. Discuss the continuity of the function:

$$f(x) = \begin{cases} 1 \ \ if\ x\ is\ rational \\ 0\ if\ x\ is\ irrational \end{cases}$$

4. Define a compact set. If K is a compact subset of R and $f: K \to R$ is continuous on K , then prove that f(K) is compact.
5. Define Riemann Stieltjes integral of a function f(x) with respect to α.
6. Show that every absolutely convergent series is convergent.
7. Distinguish between point wise and uniform convergence of a sequence of functions. Give one example for each.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Show that a set E is open if and only if its complement is closed.
9. Show that compact subsets of metric spaces are closed, and closed subsets of compact sets are compact.
10. Define differentiability of a function at a point,

    if $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x^2}\right) & if\ x \neq 0 \\ 0 & if \ \ x = 0 \end{cases}$,

    Show that f(x) is differentiable, but f' is not bounded on [-1,1].

11. Define a connected set. If f is a continuous mapping of a metric space X into a metric space Y, and if E is a connected subset of X, then prove that f(E) is connected.

(P.T.O.)

12. Define refinement of a partition. If P^* is a refinement, prove that:

$L(P^*, f, \alpha) \geq L(P.f, \alpha) and\ U(P^*, f, \alpha) \leq U(P.f, \alpha).$

13. Prove the following.
a) If f is continuous on [a,b], then $f \in R(\alpha)$ on [a,b].

b) If f is monotonic on [a,b], and α is continuous on [a,b], then $f \in R(\alpha)$ on [a,b].

14. Discuss the convergence of the series:

a) $\sum_{n=1}^{\infty} \frac{x}{n(n+1)}$ in the interval (0,b) and

b) $\sum_{n=1}^{\infty} \frac{x^n}{n^3+nx^n}, 0 < x < 1.$

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. a) Let P be a non-empty perfect set in R^k. Show that P is uncountable.

b) State and prove Bolzano Weirstrass Theorem.

16. a) State and prove the Lagrange's mean value theorem.

b) State Taylors theorem. If $f(x) = |x|^3$, compute the higher order derivatives, $f'(x)$ and $f''(x)$. What can you say about $f'''(0)$?

17. a) If $f \in R(\alpha_1)$ and $f \in R(\alpha_2)$ on [a,b], prove that:

$f \in R(\alpha_1 + \alpha_2)$ and $\int_a^b f\, d(\alpha_1 + \alpha_2) = \int_a^b f\, d\alpha_1 + \int_a^b f\, d\alpha_2.$

b) If $f \in R(\alpha)$ on [a,b], show that $|f| \in R(\alpha)$ and $\left|\int_a^b f\, d\alpha\right| \leq \int_a^b |f| d\alpha$.

18. a) Prove that if $\{f_n\}$ is a sequence of differentiable functions on [a,b], such that it converges at least at one point $x_0 \in [a, b]$ and the sequence of differentials $\{f_n'\}$ converges uniformly to G on [a,b], then $\{f_n\}$ converges uniformly to f on [a,b] and $f'(x) = G(x)$.

b) Show that $\{f_n(x)\}$, where $f_n(x) = \frac{x}{1+nx^2}$ converges uniformly to a function f, and $f'(x) = \lim_{n\to\infty} f_n'(x)$ if $x \neq 0$, but $f'(x) \neq \lim_{n\to\infty} f_n'(x)$ if x= 0.

(2 × 5 = 10 weightage)