FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C01-MEASURE THEORY AND INTEGRATION

Time: 3 Hours   **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define limsup and liminf of a sequence of sets. Furnish an example for which they are different.
2. If f is measurable function, show that f^2 is also measurable function.
3. State Holder's inequality.
4. Define integral of a non-negative measurable function.
5. State Radon Nikodym theorem.
6. What is product sigma field?
7. State Hahn Decomposition theorem.

$$(4 \times 2 = 8 \text{ weightage})$$

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Establish Monotone Convergence theorem.
9. What do you mean by integrable functions? If f is integrable and c is a real constant, then show that cf is integrable.
10. If f is measurable, g is integrable and $|f| \leq g$, then show that f is integrable and $\int |f| d\mu \leq \int |g| d\mu$.
11. State and prove Jordan decomposition theorem.
12. State Fubini's theorem.
13. If $\lambda << \mu$, $\mu << \vartheta$, then show that $\lambda << \vartheta$ and $\frac{d\lambda}{d\vartheta} = \frac{d\lambda}{d\mu}\frac{d\mu}{d\vartheta}$ a.e.
14. Define outer measure. When do you say that a measure space is complete?

$$(4 \times 3 = 12 \text{ weightage})$$

Part C: Answer any *two* questions. Each carries *five* weightage.

15. State and prove Minkowski's inequality.
16. State and prove Hahn decomposition theorem.
17. State and prove Lebesgue decomposition theorem.
18. State and prove Tonelli's theorem.

$$(2 \times 5 = 10 \text{ weightage})$$