D1AMT2304

FIRST SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH1C04-DISCRETE MATHEMATICS

Time: Three Hours

Max. Weightage : 30

Part A: Answer *all* questions. Each carries *1* weightage.

1. Define Partial order. Give example of a Partial order which is not a total order and draw the Hasse diagram.

2. Show that Associate laws hold in a Boolean Algebra.

3. Define D.N.F and find the D.N.F of $f(x_1, x_2, x_3) = x_1 x_3 + x_2 x_3' + x_1 x_2$

4. Define Composition of 2 graphs. Find $K_2[P_3]$.

5. Prove or Disprove : No loop can belong to an edge cut.

6. Define a Planar graph. Show that for a self-dual graph $2n = m + 2$.

7. Define reverse of a string and Prove that $(w^R)^R = w$ or all $w \in \Sigma^*$.

8. Define a regular language. Give Example.

$(8 \times 1 = 8$ **weightage**$)$

Part B: Answer *any two* questions from *each unit.*
Each carries *2* weightage.

Unit I

9. Define Lattice. Let $(X, +, ., ')$ be a Boolean Algebra and if $x, y \in X$, we define $x \leq y$ if $x.y' = 0$. Show that $\leq$ makes X into a lattice.

10. Define atoms in a Boolean Algebra. Show that in a finite Boolean Algebra $(X, +, ., ')$ every element of X can be expressed as a sum of atoms.

11. Prove that the characteristic numbers of a symmetric Boolean function completely determine it.

Unit II

12. Show that a connected simple graph G is 3-edge connected if, and only if, every edge of G is the exact intersection of the edge sets of two cycles of G.

13. Show that the number of edges in a tree with n vertices is $n - 1$. Whether the converse is true. Justify your answer.

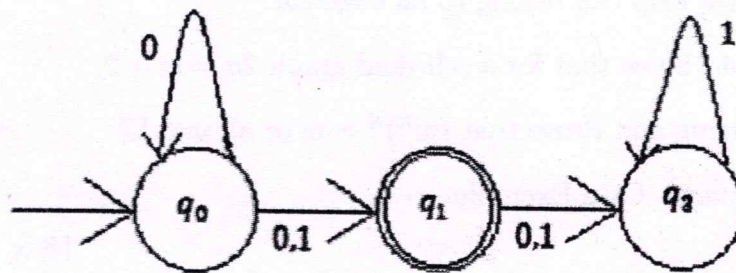14. State and prove Euler formula for connected plane graphs.

Unit III

15. Find Grammer for $\Sigma = \{a, b\}$ that generates the set of all strings with atleast one a and with exactly one a.

16. Find a dfa that accepts all strings on $\{0, 1\}$, except those containing the substring 001.

17. Find dfa and nfa accepting the language $\{(10)^n : n \geq 0\}$.

<div align="right">(6 x 2= 12 weightage)</div>

Part C: Answer any *any two* questions. Each carries 5 weightage.

18. State and prove Stone representation theorem for finite Boolean algebras.

19. State and Prove Whitney's Theorem

20. Define Eulerian Graph and Give example. Prove that a connected graph G is Eulerian if and only if G is an edge disjoint union of cycles.

21. Convert the nfa into equivalent dfa.

<div align="right">(2 x 5 = 10 weightage)</div>