D1AST2205 **(2 Pages)** **Name.............................**

Reg.No...........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C05-DISTRIBUTION THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. What is a hierarchical model? Give one example.
2. State recurrence relation for cumulants of power series distribution.
3. Let $X_k \to Geo(p_k), k = 1,2,\dots,n$ be a sequence of independent random variables and $m_n = Min(X_1, X_2, \dots, X_n)$. Show that $m_n \xrightarrow{d} Geo\ (p)$ where $p = \prod_{k=1}^{n} p_k$.
4. Define log normal distribution. Find the distribution of $Y = \prod_{i=1}^{n} X_i$, where X_i's are independent and identically distributed lognormal random variables.
5. Define order statistics. Write the formula for the probability density function of $R = X_{(n)} - X_{(1)}$.
6. If X & Y are independent binomial random variable such that $X \xrightarrow{d} B(m,p)$, and $Y \xrightarrow{d} B(n,p)$, show that $X/(X+Y)$ is hyper geometric.
7. Discuss the properties of location-scale family.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Define Chi-square distribution and state its applications. Also discuss its interrelationship with other sampling distributions.
9. If X and Y are independent exponential random variables with parameter β, show that $\frac{X}{X+Y}$ has $U(0,1)$ distribution.
10. Define Negative binomial distribution. Also derive the recurrence relation of its central moments.
11. Define the Weibull distribution and obtain it as a transformation of the exponential random variable. Also find the mean and variance of Weibull distribution.
12. Briefly explain about Pearsonian system of distributions.
13. If $X \xrightarrow{d} Beta(m,n)$, show that $Y = \frac{n}{m}\frac{X}{1-X}$ has $F(2m, 2n)$ distribution.
14. State and prove Minkowski's inequality.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. If X_1 , $X_2, \dots, X_n$ are i.i.d r.v's having $U(0, a)$ distribution. Identify the distribution of midrange $V = \frac{X_{(n)} + X_{(1)}}{2}$.
16. If (X, Y) has a bivariate normal distribution, find $E(X|Y)$& $E(Y|X)$.
17. If X_1& X_2 are independent gamma random variables with same scale parameters, show that $X_1 + X_2 and \frac{X_1}{X_1 + X_2}$ are independently distributed. Identify their distributions.
18. Derive non-central t-distribution. When will this reduce to central t distribution?

(2 × 5 = 10 weightage)