D1AST2203

Name.............................

Reg. No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022
STATISTICS
FMST1C03-ANALYTICAL TOOL FOR STATISTICS II

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage

1. Compute an orthonormal basis for the basis S = {u_1, u_2, u_3} where u_1 = (1, 1, 1), u_2 = (-1, 0, -1) and u_3 = (-1, 2, 3).
2. Define Hermitian matrix. If A is Hermitian, then show that the eigen values of A are all real numbers.
3. Show that the minimal polynomial of a matrix A exists and is unique.
4. Define inner product of two vectors. Give example.
5. Find the g – inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.
6. Define rank of a matrix. Prove that rank of the product of two matrices cannot exceed the rank of either matrix.
7. Find the Jordan canonical form of $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage

8. (a) Define a subspace. Let $W = \{\{x, y, z\}; x^2 = y\}$. Is W a subspace of $\mathbb{R}^3$?
   (b) Prove that the intersection of two subspaces of a vector space V is a subspace of V.
9. Show that any two characteristic vectors corresponding to two distinct characteristic roots of a symmetric matrix are orthogonal.
10. a) Define basis and dimension.
    b) Determine whether or not the following set is a basis for $\mathbb{R}^4$.
    {(1, −1 ,1, -1), (2, 2, 2, 2), (1,2,1,2), (1, 2, 3, 4)}.
11. Describe the method of finding the inverse of a non-singular matrix A by forming a partition of A.
12. Define algebraic multiplicity and geometric multiplicity of characteristic roots. Show that the geometric multiplicity of a characteristic root cannot exceed algebraic multiplicity of the same.
13. a) Prove that all bases of a finite-dimensional vector space over F have the same number of elements
    b) Let V be a finite dimensional vector space and let {$v_1, v_2, ..., v_n$} be any basis. Prove that if a set has more than n vectors, then it is linearly dependent
14. Define Moore Penrose inverse of a matrix. Show that it is unique.

b) Find Moore Penrose inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix}$.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage

15. a) Let V be of finite dimension, and let $T: V \to U$ be linear. Prove that rank (T)+ nullity (T) = dim V.

b) Find the rank and nullity of the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by T(x, y, z) = (x+2y - z, y + z, x + y-2z).

16. a) State and prove Cayley Hamilton Theorem.

b) Using Cayley –Hamilton theorem obtain the inverse of the matrix $\begin{bmatrix} 6 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

17. a) Illustrate the different types of quadratic forms and state a condition characterizing each type of the quadratic form.

b) Reduce the quadratic form $5x^2 + y^2 + 10z^2 - 4yz - 10zx$ to its canonical form and discuss its definiteness.

18. Define direct sum and compliment of a subspace.

a) Prove that V=U⊕W if and only if (i) V=U+W and (ii) U∩W={0}

b)Let U and W be the following subspaces of $\mathbb{R}^3$:

U={(a, b, c): a = b = c} and W={(0,b,c)}. Show that $\mathbb{R}^3$=U⊕W.

(2 × 5 = 10 weightage)