D1AST2202 **(2 Page)** **Name............................**
Reg.No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C02-ANALTYCAL TOOL FOR STATISTICS I

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define: (i) Perfect set; (ii) Compact set; (ii) Connected set.
2. Discuss whether the set of rational numbers and set of real numbers are countable or not.
3. Define limit of a real valued function. Evaluate $\lim_{x\to 0} \frac{|x|}{x}$.
4. If f is a continuous function from a compact metric space to another metric space, show that f is uniformly continuous.
5. Define the terms: (i) norm; (ii) partition and (ii) refinement. Illustrate all these with the help of examples.
6. Define point wise and uniform convergence of a sequence of functions. State the mutual relationships between these two.
7. Examine the uniform convergence of $\{1-(1-x^2)^n\}$ in (-1/2, 1/2).

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Define a metric space. If (X,d) is a metric space, show that the function defined by

   $d_1(x,y) = \frac{d(x,y)}{1+d(x.y)}\ for\ all\ x, y\ \in X$ is a metric on X.

9. Define continuity and derivability of a real valued function. Establish the mutual implications between them.
10. a) Find $\lim_{x\to 0} \frac{\sin x}{\sqrt{x}}$

    b) Define derivative of a real valued function and find the derivative of the function $f(x) = \begin{cases} 2 & if \quad x \le 1 \\ x & if \quad x > 1 \end{cases}$ at x= 1.

11. Define Riemann Stieltjes integral and evaluate $\int_0^5 f\ d\alpha(x)$ when $f(x) = x^2$ and

    $\alpha(x) = [x] - x,$ (where $[x]$ is the integer part of x).

(P.T.O.)

12. Prove that if $f \in R(\alpha)$ and $g \in R(\alpha)$, then $fg \in R(\alpha)$.

13. Establish Cauchys criterion of uniform convergence of sequence of real numbers.

14. Discuss the uniform convergence of $\{f_n\}$ where $f_n(x) = e^{-nx}$ for $x \geq 0$.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. a) Show that the union of two connected sets A *and* B is connected, $where\ A \cap B \neq \phi$.

    b) Prove that every infinite bounded set has a limit point.

16. a) State and prove Lagrange's mean value theorem. Examine the theorem for the function f(x)= log x on [1/2,2].

    b) Establish the statement; Lagrange's mean value theorem is a particular case of Cauchy's mean value theorem.

17. a) Prove that, a function f is integrable with respect to α if and only if for every $\epsilon > 0$, there exists a partition P of [a,b], such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$.

    b) Show that if $f \in R(\alpha)$ and $f \in R(\beta)$ on [a,b], then $f \in R(\alpha + \beta)$ on [a,b].

18. a) Prove the following: Let $\sum f_n$ be a series of differentiable functions on [a,b] and it converges at least at one point x_0 in [a,b]. If the series of differentials $\sum f_n{}'$ converges uniformly to G on [a,b] then $\sum f_n$ converges uniformly to f on [a,b], where $f'(x) = G(x)$.

    b) Show that the series $1 - x + x^2 - \cdots, 0 \leq x \leq 1$, admits term by term integration on [0,1], though it is not uniformly convergent.

(2 × 5 = 10 weightage)