D1AST2201 **(2 Pages)** **Name.............................**
Reg.No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C01-MEASURE THEORY AND INTEGRATION

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Let μ be a measure defined on a σ algebra ***X***. If (F_n) is a decreasing sequence in ***X*** and if $\mu(F_1) < \infty$, then show that $\mu(\bigcap_{n=1}^{\infty} F_n) = \lim \mu(F_n)$.

2. If (μ_n) is a sequence of measures on ***X*** with $\mu(\boldsymbol{X}) = 1$ and if λ is defined by $\lambda(E) = \sum_{n=1}^{\infty} 2^{-n} \mu_n(E)$, E in ***X*** , show that λ is a measure on ***X*** and $\lambda(\boldsymbol{X}) = 1$.

3. If f is measurable and B is a Borel set, then show that $f^{-1}(B)$ is measurable.

4. If f is a measurable function and if f = g a.e, show that g is measurable.

5. Suppose that f belongs to M^+. Then prove that f(x) = 0 μ-almost everywhere on ***X*** if and only if $\int f \, d\mu = 0$.

6. Establish the uniqueness of the Radon-Nykodym derivative.

7. If μ is a signed measure such that $\mu \perp \lambda$ and $\mu \ll \lambda$ then show that μ =0.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Let f : X → Y be a function and let C and D be subsets of the co-domain Y .
   Then prove:
   a) $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$
   b) $f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D)$.
9. If f and g are integrable over E , show that f + g is integarble over E and $\int_E (f+g) = \int_E f + \int_E g$
10. State and prove Lebesgue dominated convergence theorem.
11. Establish Minkowski's inequality.
12. Let λ, μ and ν be σ-finite measures on (X, ***X***). If $\nu \ll \mu \ll \lambda$

    Then show that $\frac{d\nu}{d\lambda} = \frac{d\nu}{d\mu} \frac{d\mu}{d\lambda}$.
13. If $f_n \to f$ in measure show that there is a subsequence $\{ f_{nk} \}$ which converge to f a.e.
14. Define convergence in L_p. Give an example of sequence (f_n) of functions which converges in L_p but does not converge to f (x) for any x in X.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Under usual assumptions, prove that a non-negative function is the limit of a sequence of a monotone increasing sequence of simple functions.
16. State and prove Fatous' lemma and use it to prove the Monotone Convergence theorem.
17. State and prove product measure theorem.
18. State and prove Lebesgue decomposition theorem. Mention the application of this result in Probability theory.

(2 × 5 = 10 weightage)