D1APH2202 **(2 Pages)** **Name.....**
Reg.No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(Regular/Improvement/Supplementary)

PHYSICS

FPHY1C02- MATHEMATICAL PHYSICS I

Time: 3 Hours **Maximum Weightage: 30**

Part A: Short answer questions. Answer *all* questions. Each carries *one* weightage.

1. Analyze the basic features of Cylindrical Coordinate System.
2. Show that Laplace transform is a linear operation.
3. Explain how separation of variables method used to find out solution of partial differential equations of Physics.
4. What is meant by singular point of a differential equation?
5. Show that $\beta(m,n) = \beta(n,m)$.
6. Show that $L_n(0) = 1$.
7. Write the Dirichlet conditions for Fourier series.
8. Give a short note on Inverse Fourier Transform.

(8 × 1 = 8 weightage)

Part B: Essay questions. Answer any *two* questions. Each carries *five* weightage.

9. Formulate the general expression for gradient, divergence and Curl in Orthogonal curvilinear coordinate system.
10. Define tensors. Explain Contraction, Direct product and Quotient rule of tensors.
11. Discuss the Frobenius method for obtaining solution of a linear, second order homogeneous Ordinary Differential Equation. Using this method find out series solution of $\frac{d^2y}{dx^2} + \omega^2 y = 0$.
12. Solve Bessel's differential equation of order n.

(2 × 5 = 10 weightage)

(P.T.O.)

Part C: Problems. Answer any *four* questions. Each carries *three* weightage.

13. Define Orthogonal, Hermitian and Unitary matrices. Give one example for each.

14. What is Hermitian Operators? Show that the eigenvalues of a Hermitian operator are real.

15. Prove that $\Gamma(n) = 2\int_0^\infty e^{-(y^2)}\ y^{2n-1}\ dy$ and find out the value of $\Gamma\left(\frac{1}{2}\right)$.

16. Prove the following relations:

    a) $P_n(1) = 1.$

    b) $P_{2n}(0) = (-1)^n\ \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots2n}$

17. Prove the recurrence relation for Hermite polynomial:

$$H_{n+1}(x) = 2x\ H_n(x) - 2n\ H_{n-1}(x)$$

18. Find the Fourier series for $f(x) = x^2$, for $-\pi \le x \le \pi$. Hence show that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

19. Find the Laplace transform of $F(t)$ where

$$F(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right) & ; \quad t > \frac{2\pi}{3} \\ \\ 0 & ; \quad t < \frac{2\pi}{3} \end{cases}$$

(4 × 3 = 12 weightage)