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FIRST SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Regular/Improvement/Supplementary)

MATHEMATICS
FMTH1C04- DISCRETE MATHEMATICS

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries 1 weightage.

1. Give an example of a partial order on a set which is not a total order.
2. State and prove laws of tautology in a boolean algebra.
3. Define self complementary graphs and give one example.
4. Give an example of a non simple connected graph with $\delta \geq \frac{n-1}{2}$
5. For any simple graph prove that $Aut(G) = Aut(G^c)$
6. Give an example of a graph with n vertices and $n-1$ edges that is not a tree.
7. Use induction on n to show that $|u^n| = n|u|$ for all strings u and all n.
8. Prove that $(\omega^R)^R = \omega$ for all $\omega \in \sum^*$.

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries 2 weightage.

Unit 1

9. a) Let R be an equivalence relation on a set X. Then for every $x \in X$, prove that $x \in [x]$. Also for any $x, y \in X$, either $[x] = [y]$ or $[x] \cap [y] = \phi$.

   b) If $x_1, x_2, \ldots\ldots x_n$ are elements of a boolean algebra then prove that $x_1 + x_2 + \ldots\ldots + x_n = 0$ if and only if $x_i = 0$ for all $i = 1$ to n.

10. a) Let $(X, \leq)$ be a poset and A, a non empty finite subset of X. Then prove that A has at least one maximal element. Also prove that A has a maximum element if and only if it has a unique maximal element.

    b) Prove that a well ordered set is totally ordered.

11. a) Prove that a partial order $\leq$ on a set X is total if and only if the corresponding strict order $<$ satisfies the following property: for all $x, y \in X$, either $x < y$ or $x = y$ or $y < x$.

b) Let Y be a sub algebra of a boolean algebra $(X,+\bullet)$. Then prove that Y is a boolean algebra.

Unit 2

12. a) If $\{x, y\}$ is a 2-edge cut of a graph G, show that every cycle of G contains x must also contain y.

b) An edge $e = xy$ of a connected graph G is a cut edge of G if and only if e belongs to no cycle of G.

13. a) If $\delta(G) \geq 2$, G contains a cycle.

b) A connected graph G is a tree if and only if every edge of G is a cut edge of G.

14. a) A graph is planar if and only if it is embedded on a sphere.

b) Let G be a plane graph and f be a face of G. Then there exists a plane embedding of G in which f is the exterior face.

Unit 3

15. a) Prove or disprove $(L_1 \cup L_2)^R = L_1^R \cup L_2^R$ for all languages L_1 and L_2.

b) Find a grammar that generates the language $L = \{\omega\omega^R : \omega \in \{a,b\}^+\}$. Give a complete justification of your answer..

16. Let $M = (Q, E, \delta, q_o, F)$ be a dfa, and let G_M be its associated transition graph. Then for every $q_i, q_j \in Q$ and $w \in E^+, \delta^*(q_j, w) = q_j$ if and only if there is in G_M a walk with label w from q_i to q_j.

17. Find a dfa that accepts all strings on $\{0,1\}$, except those containing the sub string 001

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage.

18. a) Every finite boolean algebra is isomorphic to a power set boolean algebra.

b) Write the following boolean function in disjunctive normal form.

$$f(x_1, x_2, x_3) = (x_1 + x_2^1)x_3^1 + x_2 x_1^1 (x_2 + x_1^1 x_3).$$

19. a) Prove that the number of edges in a tree on n vertices is $n-1$. Conversely prove that a connected graph on n vertices and $n-1$ edges is a tree.

b) Every connected graph contains a spanning tree.

20. a) Prove that $K_{3,3}$ is non planar.

b) State and prove Euler's formula in plane graphs.

21. a) Define an nfa with no more than five states for the set

$$\{abab^n : n > 0\} \cup \{aba^n : n \geq 0\}$$

b) Let L be the language accepted by an nfa $M_N = (Q_N, E, \delta_N, q_0, F_N)$.

Then prove that there exists a dfa $M_D = (Q_D, E, \delta_D, \{q_0\}, F_D)$ such that $L = L(M_D)$.

(2 × 5 = 10 weightage)