D1AMT2203 **(2 Pages)** **Name......................**

Reg.No...................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(Regular/Improvement/Supplementary)

MATHEMATICS

FMTH1C03 - REAL ANALYSIS I

Time: 3 Hours **Maximum weightage: 30**

Part A

Answer *all* questions. Each carries 1 weightage.

1. Define countable set. Let $A_i, i = 1, 2, 3\ldots$ be a sequence of countable sets and let $A = \{(a_1, a_2, a_3 \ldots\ldots) \; / \; a_i \in A_i, i = 1, 2, 3, \ldots\}$. Is the set A always countable? Justify your answer.

2. Define Compact Set. Show that every finite sets are compact in any metric space.

3. Let f be a real continuous function on a metric space X. Let $E = \{x \in X : f(x) = 0\}$. Show that E is a closed subset of X.

4. Give an example of differentiable function f whose derivative f' is not continuous. Justify your answer.

5. Suppose $f \geq 0$, f is continuous on $[0, 1]$, and $\int_0^1 f(x)dx = 0$. Prove that $f(x) = 0$ for all $x \in [0, 1]$.

6. If $f \in \mathcal{R}(\alpha)$, $g \in \mathcal{R}(\alpha)$, and c_1, c_2 are constants. Then show that $c_1 f + c_2 g \in \mathcal{R}(\alpha)$.

7. Let $f_n(x) = x^n$ be the sequence of functions defined on $[0, a]$, $a > 0$. For what values of a the sequence converges uniformly ? Justify your answer.

8. Construct sequences $\{f_n\}, \{g_n\}$ which converges uniformly on some set E, but the sequence $\{f_n g_n\}$ does not converge uniformly on E.

(8 x 1= 8 weightage)

Part B

Answer any *two* questions from each unit. Each carries 2 weightage.

Unit 1

9. Show that a set E is open if and only if its complement is closed.

10. Define closure of E. Show that $\bar{E}$ is the smallest closed subset of X that contains E.

11. Let f be a continuous mapping of a compact metric space X into a metric space Y. Then show that f is uniformly continuous.

(P.T.O.)

Unit 2

12. State and prove generalized mean value theorem.

13. If f is differentiable on $[a, b]$, then show that f' cannot have any simple discontinuities on $[a, b]$.

14. If f is continuous on $[a, b]$ then show that $f \in \mathcal{R}(\alpha)$ on $[a, b]$.

Unit 3

15. If γ' is continuous on $[a, b]$, then show that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|dt$.

16. Show that there exists a real continuous function on the real line which is nowhere differentiable.

17. Suppose $\{f_n\}$ is a sequence of functions defined on E, and suppose $|f_n(x)| \leq M_n \ (x \in E, n = 1, 2, 3......)$ Then show that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.

(6 x 2= 12 weightage)

Part C

Answer any *two* questions. Each carries 5 weightage.

18. a) Show that every $k-$cell is compact.

    b) A mapping of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(C)$ is closed in X for every closed set C in Y.

19. a)If P^* is a refinement of P then show that $U(P^*, f, \alpha) \leq U(P, f, \alpha)$

    b) State and prove fundamental theorem of calculus.

20. a) If f_n is a sequence of continuous functions on E, and if $f_n \to f$ uniformly on E, then show that f is continuous on E.

    b) If K is compact, if $f_n \in \mathcal{C}(K)$ for $n = 1, 2, 3, ...$ and if $\{f_n\}$ is a pointwise bounded and equicontinuous on K then show that $\{f_n\}$ contains a uniformly convergent subsequence.

21. State and prove Stone- Weierstrass Theorem.

(2 x 5= 10 weightage)