D1AST2105 **(2 Pages)** **Name............................**
Reg.No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C05-DISTRIBUTION THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. If X and Y are independent Poisson random variables with respective means λ_1 and λ_2. Find $E\left(X/_{X+Y=n}\right)$.
2. If X and Y are independent exponential random variables with parameter β, show that $\frac{X}{X+Y}$ has $U(0,1)$ distribution.
3. For any integer valued random variable, show that $\sum_{n=0}^{\infty} s^n P(X \le n) = (1-s)^n P(s)$, where P is the PGF of X.
4. Show that $Var(X) = E\big(Var(X|Y)\big) + Var\big(E(X|Y)\big)$.
5. Derive the joint distribution of $X_{(r)}, X_{(s)}$, the r^{th} and s^{th} order statistic.
6. Explain location-scale family of distributions. Give examples.
7. If $X_1, X_2, \ldots, X_n$ are $U(0,1)$ random variables, show that $nMin(X_1, X_2, \ldots, X_n)$ is asymptotically exponentially distributed.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Derive the distribution of $R = X_{(r)} - X_{(s)}$, if $X_1, X_2, \ldots, X_n$ are independent and identically distributed $U(0,1)$ random variables.
9. Define log-Normal distribution. Find its characteristic function.
10. In sampling from normal distribution, show that the sample mean $\bar{X}$ and sample variance S^2 are independent.
11. If $X \overset{d}{\rightarrow} Cauchy\ (0,1)$, then show that its moment generating function does not exist. Also find the probability distribution of $Y = \frac{1}{X}$
12. If $X \overset{d}{\rightarrow} Beta\ (m,n)$, show that $\frac{n}{m}\frac{X}{1-X}$ has $F(2m, 2n)$ distribution.
13. Define Hypergeometric distribution. Find its mean and variance.

(P.T.O.)

14. Let X be a random variable with a continuous distribution function F. Then show that $Y = F(X)$ has uniform distribution on $[0,1]$.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Define power series distribution. Explain how we obtain binomial distribution from power series distribution. Also identify the expression for moment generating function, mean and variance of power series distribution.

16. i) Explain about Weibull distribution. Show that $Min(X_1, X_2, \dots, X_n)$ follows Weibull distribution if and only if X_i's are Weibull distributed.

    ii) State and prove Holder's inequality.

17. If X is a non negative random variable with distribution function $F(.)$, show that $E(X) = \int_0^\infty \left(1 - F(x)\right) dx$. Using this formula, identify $E(X)$ of exponential distribution with parameter β.

18. Derive probability density function of non central F distribution.

(2 × 5 = 10 weightage)