D1AST2103 **(2 Pages)** **Name: ....**

Reg. No.: ....

FIRST SEMESETER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021
(Regular/Improvement/Supplementary)

STATISTICS

FMST1C03 - ANALYTICAL TOOLS FOR STATISTICS-II

Time: 3 **hours** **Maximum weightage:** 30

Part A

Answer any* four *questions.
Each question carries two weightage

1. What do you mean by basis and dimension of a vector space? Suggest a basis of the vector space R^3.
2. What do you mean by a set of orthogonal vectors?. Show that orthogonal vectors are linearly independent but the converse need not be true.
3. Define rank factorization of a matrix and illustrate with an example.
4. Define idempotent matrix and nilpotent matrix and give an example in each case.
5. If λ is a characteristic root of a square matrix show that λ^2 is a characteristic root of A^2.
6. Define Moore-Penrose inverse of a matrix. Obtain the Moore-Penrose inverse of
$$A = \begin{pmatrix} -1 & 2 \\ 1 & -2 \end{pmatrix}$$
7. Distinguish between a positive definite quadratic form and positive semi-definite quadratic form. Give an example in each case.

(4 x 2= 8 weightage)

Part B

Answer any* four *questions.
Each question carries* three *weightage.

8. What do you mean by linear independence of a set of vectors? Examine whether the vectors V_1, V_2, V_3 are linearly independent,where $V_1 = (-2, -2, 4, 0), V_2 = (2, 4, 6, 8)$, $V_3 = (1, 0, 5, -3)$
9. What do you mean by subspace of a vector space? Consider the vector space R^2. Examine whether the subsets S_1, S_2 and S_3 of R^2 are subspaces, where $S_1 = \{(x, -2x)\}$, $S_2 = \{(x, 2x - 3)\}$ and $S_3 = \{(x, 3x^3)\}$.

(P.T.O.)

10. Define row space and column space of a matrix. If A and B are two matrices such that AB exists, then show that $rank\,(AB) \leq min\{rank(A),\, rank(B)\}$.

11. If A is a square matrix, show that the sum of the characteristic roots is the trace of A and the product of the characteristic roots is equal to $|A|$.

12. Determine the characteristic roots and characteristic vectors of the matrix $\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$

13. Define g-inverse of a matrix and show that GA is an idempotent matrix if G is a g-inverse of A. Find a g-inverse of the following matrix

$$\begin{pmatrix} 3 & -1 & 2 \\ 2 & 0 & -4 \end{pmatrix}$$

14. Determine the definiteness of the following quadratic form

$$Q = 2x^2 + y^2 + 5z^2 - xy + 2xz - 4yz$$

(4 x 3= 12 weightage)

Part C

Answer any* two *questions. Each question carries* five *weightage.

15. (a) Examine whether the set of all 2×2 matrices with real numbers as elements is a vector subspace. If so obtain a basis of it.

    (b) For any two subspaces S_1 and S_2 show that $\dim(S_1 + S_2) + \dim(S_1 \cap S_2) = \dim(S_1) + \dim(S_2)$.

16. (a) State and prove rank nullity theorem.

    (b) Define row rank and column rank of a matrix and show that they are equal.

17. (a) State and prove Cayley-Hamilton theorem.

    (b) Find the eigen values and eigen vectors of the following matrix

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 3 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

18. (a) Show that when a matrix A is symmetric, the eigen vectors corresponding to the distinct eigen values are orthogonal to each other.

    (b) Define rank and signature of a real quadratic form and obtain them for the following quadratic form

$$Q = 6x_1^2 + 2x_2^2 - 8x_3^2 + 4x_1x_2 + 8x_2x_3 - 6x_3x_1$$

.

.

(2 x 5= 10 weightage)