D1AST2101 (1 Page) Name.............................
Reg.No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C01-MEASURE THEORY AND INTEGRATION

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage

1. Define a field and a sigma field. Show by an example that union of two sigma fields need not be a sigma field.
2. What are the axioms of measure? Distinguish between finite measure and sigma finite measure.
3. What do you mean by normed linear space?
4. State Minkowski's inequality.
5. State Radon Nikodym Theorem.
6. Distinguish between Lebesgue and Lebesgue Stieltjes measure.
7. State Caratheodory extension theorem.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. State and prove Fatou's lemma.
9. If f and g are measurable, show that f + g is also measurable.
10. State and prove Lebesgue dominated convergence theorem.
11. Check whether L_p convergence implies convergence in measure.
12. If f and g are simple non negative measurable functions show that,
    $$\int (f+g)\, d\mu = \int f\, d\mu + \int g\, d\mu.$$
13. Distinguish between almost everywhere convergence and almost uniform convergence.
14. What do you mean by product measures? State Tonelli's theorem.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. State and prove Holder's Inequality.
16. State and prove Jordan Decomposition Theorem.
17. State and prove Fubini's theorem.
18. If f is a nonnegative function in $M(X, \aleph)$, then show that there exists a nonnegative nondecreasing sequence of simple functions $\{\varphi_n\}$ such that $f(x) = lim\varphi_n(x)$ for each $x \epsilon X$.

(2 × 5 = 10 weightage)