D1APH2102 **(2 Pages)** **Name..............................**
Reg.No...........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(Regular/Improvement/Supplementary)

PHYSICS

FPHY1C02-MATHEMATICAL PHYSICS I

Time: 3 Hours **Maximum Weightage: 30**

Part A: Short answer questions. Answer *all* questions. Each carries *one* weightage.

1. Categorize two kinds of singularities of a differential equation.
2. What is a Hermitian operator? State the properties of this operator.
3. Define Γ function. By direct integration show that $\Gamma(n+1) = n\,\Gamma(n)$
4. Prove that $H_n(-x) = (-1)^n H_n(x)$
5. List out the uses of Fourier series.
6. What are Bessel function and Spherical Bessel function?
7. Explain Fourier Transform.
8. Write down the Fourier cosine transform and Fourier sine transform.

(8 × 1 = 8 weightage)

Part B: Essay questions. Answer any *two* questions. Each carries *five* weightage.

9. Discuss Orthogonal curvilinear coordinates. Generate an expression for Gradient and Divergence in this system.
10. Discuss the method of diagonalization of matrices with example.
11. Describe the Gram–Schmidt Orthogonalization procedure with example.
12. Derive the orthogonality relation for Legendre polynomial $P_n(x)$.

(2 × 5 = 10 weightage)

(P.T.O.)

Part C: Problems. Answer any *four* questions. Each carries *three* weightage.

13. Define contravariant, covariant and mixed tensors. Give examples.

14. Discuss the Completeness of Eigen functions.

15. Show that $\beta(m,n) = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}} dy$

16. Prove the recurrence relation for Bessel function

$$J_{n+1}(x) + J_{n-1}(x) = \frac{2n}{x} J_n(x)$$

17. Show that $(1 - 2xz + z^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(x) Z^n$

18. Show that the function $f(x) = \begin{cases} 0 \;\; for \; -\pi \leq x < 0 \\ x \;\; for \;\; 0 \leq x < \pi \end{cases}$ can be expanded in Fourier series as

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)^2} - \sum_{n=1}^{\infty} (-1)^n \frac{\sin nx}{n}$$

19. Find the Laplace transform of the function $F(t)$ where

$$F(t) = \begin{cases} \cos t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$$

(4 × 3 = 12 weightage)