**D1AMT2104**

**FIRST SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2021**
**MATHEMATICS**
**FMTH1C04-DISCRETE MATHEMATICS**

**Time: Three Hours**                                    Max. Weightage : 30

**Part A**
Answer **all** questions. Each carries 1 weightage.

1. Give example of a lattice on the set $X = \{a, b, c\}$.

2. In a Boolean algebra $(X, +, \cdot)$, show that $(x + y)' = x' \cdot y'$.

3. Draw a graph on four vertices and give its complement.

4. Is it possible to draw a graph having graphical sequence $\{1, 2, 3, 4, 5\}$ ?
   Justify your claim.

5. Define a connected graph. Give one example.

6. Define Eulerian graph. Give example of a complete bipartite graph that is Eulerian.

7. Define an automata.

8. Find the grammar that generates the language $\{a^{n+1}b^n, n \geq 0\}$.

                                            (8 x 1= 8 weightage)

**Part B**
Answer any **two** questions from each unit.
Each carries 2 weightage.

Unit I

9.  (a) Define a partial order, minimal element and the minimum element. Give one
        example for each.

    (b) Is it possible to have a poset on a non-empty set with more than one maximal
        element ? If YES, give one example.

10. (a) Define the characteristic number of a Boolean function and give one example.

    (b) Define isomorphism of Boolean algebras and give one example.

11. (a) Let $(X, \leq)$ be a poset and $x \in X$. Let $A = \{z \in X : x < z\}$. Prove that an
        element $y \in X$ covers $x$ if and only if $y$ is a minimal element of $A$.

    (b) How do you convert a Boolean function from the conjunctive normal form to
        disjunctive normal form ?

                                                        (P.T.O.)

## Unit II

12. (a) Define cut vertex and cut edge in a graph. Give one example for each.

    (b) Can you draw a graph with an odd number of vertices in which all vertices are of odd degree ? Give reason.

13. (a) Describe the method of finding a spanning tree for a connected graph.

    (b) Explain a real life application of graph theory.

14. (a) State and prove Euler's formula.

    (b) Describe a method to check a graph to be Eulerian.
Illustrate it with an example.

## Unit III

15. (a) What is concatenation of strings ? How is it useful in a formal language ?

    (b) Explain with an example the method of getting a grammar using productions.

16. (a) Differentiate between deterministic and non-deterministic finite automata.

    (b) Give one example for each of these two types of automata.

17. (a) Show that $\{aab\}$ is a language on an alphabet $\Sigma = \{a, b\}$.
Get an *nfa* accepting this language.

    (b) Obtain a *dfa* accepting the language $\{abb\}$ on an alphabet $\Sigma = \{a, b\}$

$$(6 \times 2 = 12 \text{ Weightage})$$

## Part C
Answer any **two** questions. Each carries 5 weightage

18. (a) Define a partial order relation. Give example of one such relation on the set of all positive integers less than 100. Verify it to be a partial order.

    (b) Prove that every finite Boolean algebra is isomorphic to a power set Boolean algebra.

19. (a) Define a planar graph. State and prove a characterisation theorem for planar graphs.

    (b) Describe the Konigsberg bridge problem. How do you relate this problem with a topic in graph theory ?

20. (a) State and prove a characterisation theorem for a graph to be bipartite.

    (b) Prove that a simple graph is planar if it does not contain $K_5$ or $K_{3,3}$.

21. (a) Explain the procedure of generating a language from a *dfa*.

    (b) Is it possible to obtain the same language using a *dfa* and an *nfa* ?
Explain with an example.

$$(2 \times 5 = 10 \text{ Weightage })$$