D1AMT2103

Reg. No.............................
Name: ....

FIRST SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2021
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH1C03 - REAL ANALYSIS I

Time: Three Hours | Max. Weightage : 30

Part A

Answer **all** questions. Each carries 1 weightage.

1. Define countable set. Show that every infinite subset of a countable set A is countable.
2. Prove or disprove: Let $\{G_n\}_{n=1}^{\infty}$ be an infinite collection of open sets, then $G = \cap_{n=1}^{\infty} G_n$ is open.
3. Define discontinuity of second kind. Give example of a function that has a discontinuity of second kind.
4. Show that if f is differentiable at $x \in (a, b)$ then f is continuous at x. Whether the converse is true? Justify.
5. Show that if f is differentiable on $[a, b]$, then f' cannot have any simple discontinuity on $[a, b]$.
6. Show that if f is continuous on $[a, b]$, then $f \in \mathcal{R}(\alpha)$ on $[a, b]$.
7. Prove that if f maps $[a, b]$ into $\mathbb{R}^k$ and if $f \in \mathcal{R}(\alpha)$ for some monotonically increasing function α on $[a, b]$, then $|f| \in \mathcal{R}(\alpha)$ and $| \int_a^b f d\alpha| \leq \int_a^b |f| d\alpha$.
8. State whether every convergent sequence of functions contains a uniformly convergent sub-sequence. Justify.

(8 x 1 = 8 Weightage)

Part B

Answer any **two** questions from each unit.
Each carries 2 weightage.

Unit I

9. Prove that a finite point set has no limit points.
10. Prove that a mapping f of a metric space X into metric space Y is continuous on X if and only if $f^{-1}(C)$ is closed in X for every closed set C in Y.
11. Show that monotonic functions have no discontinuities of second kind.

(P.T.O.)

Unit II

12. State and prove generalized mean value theorem.
13. Suppose f is a continuous mapping of $[a, b]$ onto $\mathbb{R}^k$ and f is differentiable in (a, b). Show that there exists an $x \in (a, b)$ such that $|f(b) - f(a)| \leq (b - a)|f'(x)|$.
14. Let f be monotonic on $[a, b]$ and α is monotonic and continuous on $[a, b]$. Show that $f \in \mathcal{R}(\alpha)$.

Unit III

15. State and prove Cauchy criterion for uniform convergence of functions.
16. Show that if γ' is continuous on $[a, b]$, then γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|dt$.
17. Show that if $\{f_n\}$ is a sequence of continuous function on E and if $f_n \to f$ uniformly on E, then f is continuous on E.

(6 x 2 = 12 Weightage)

Part C

Answer any **two** questions. Each carries 5 weightage

18. (a) Define uniformly continuous functions. Give example.
    (b) Show that if f is a continuous mapping of a compact metric space X into metric space Y. Then f is uniformly continuous on X.
19. (a) Define Riemann-Stieltjes integral. Give example of a Riemann-Stieltjes integrable function.
    (b) Suppose α increases monotonically and $\alpha' \in \mathcal{R}$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then show that $f \in \mathcal{R}(\alpha)$ if and only if $f\alpha' \in \mathcal{R}$ and $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)dx$.
20. (a) Let $f \in \mathcal{R}$ on $[a, b]$. For $a \leq x \leq b$. Define $F(x) = \int_a^x f(t)dt$. Then show that F is continuous on $[a, b]$. And if f is continuous at $x_0 \in [a, b]$ then show that F is differentiable at x_0 and $F'(x_0) = f(x_0)$.
    (b) Show that if $f \in \mathcal{R}$ on $[a, b]$ and if there is a differentiable function F on $[a, b]$ such that $F' = f$, then $\int_a^b f(x)dx = F(b) - F(a)$.
21. (a) Show that there exists a nowhere continuous function.
    (b) Show that there exists a continuous function on the real line which is nowhere differentiable.

(2 x 5 = 10 Weightage)